Student understanding of first order *RC* filters

Pieter Coppens*

KU Leuven, Faculty of Engineering Technology, Leuven, Belgium

Leuven Engineering and Science Education Center (LESEC)†

Johan Van den Bossche

KU Leuven, Faculty of Engineering Technology,

Technology Campus Ghent, Ghent, Belgium

Leuven Engineering and Science Education Center (LESEC)

Mieke De Cock

KU Leuven, Department of Physics and Astronomy, Leuven, Belgium

Leuven Engineering and Science Education Center (LESEC)

(Dated: January 7, 2017)

Abstract

A series of interviews with second year electronics engineering students showed several problems with understanding first order RC filters. To better explore how widespread these problems are, a questionnaire was administered to over 150 students in Belgium. One question asked to rank the output voltage of a low-pass filter with an AC or DC input signal while a second asked to rank the output voltages of a high-pass filter with doubled or halved resistor and capacitor values. In addition to a discussion of the rankings and students' consistency, the results are compared to the most common reasoning patterns students used to explain their rankings. Despite lecture and laboratory instruction, students not only rarely recognize the circuits as filters, but also fail to correctly apply Kirchhoff's laws and Ohm's law to arrive at a correct answer.

I. INTRODUCTION

In engineering education, students generally encounter electrical circuits for the first time in an introductory physics course. They learn how to calculate specific currents and voltages, using Kirchhoff's laws and Ohm's law. The focus in such an introductory course is often on DC circuits with frequency-independent components (ideal batteries, wires and resistors). The first frequency-dependent component students typically encounter is a capacitor. It is introduced in terms of charging and discharging in a circuit with a resistor and capacitor in series, analyzed in the time domain. However, in introductory electronics courses, students learn how to look at circuits from a different point of view. These courses typically start by analyzing the same RC circuit, but now in terms of filtering, input and output voltage and cutoff frequency. The analysis is done in the frequency domain and input signals are AC voltages instead of DC voltages. In this paper, we focus on student understanding of and reasoning about such first order passive RC filters because they are at the transition from DC (about which a lot of research exists) to AC circuits and the transition from time-domain behavior to frequency-domain behavior. As such, they allow the introduction of typical electronics topics via a familiar circuit. When referring to a 'filter' in the remainder of the article, a first order RC filter is meant, unless specified otherwise.

This study is part of a bigger research project about student learning in introductory electronics laboratories. Here, we report on student answers to two questions on a questionnaire related to RC filters. The context and methods are explained in section III, while the questions and their analysis are in sections IV and V. The most important findings are summarized in the conclusion of section VI, together with implications for teaching. But before that, we will start with a short literature overview in Section II.

II. LITERATURE OVERVIEW

Over the past decades, a lot of research has been done on student understanding of basic electric circuits, we refer to the literature review in Zavala's thesis for an excellent overview.[1] Most of this research focused on so-called 'bulbs and batteries' questions, probing student understanding of current and voltage as well as Kirchhoff's laws and Ohm's law in passive DC circuits. Picciarelli, Di Gennaro, Stella and Conte for example found that students struggle with interpreting Ohm's law: they think that current causes a potential difference instead of the other way around. For instance, some students think that when adding a resistor parallel to an existing one, the total current will increase (correct), which consequently causes the potential difference across the original resistor to increase as well (incorrect).[2] Holton, Verma and Biswas observed problems with Kirchoff's current law (KCL), which are similar to those encountered by Engelhardt and Beichner.[3,4] Students for example think that at a node, current is always split equally between the different branches, regardless of the impedance of each branch. This is also referred to as so-called local reasoning.

Besides problems with the laws as such, McDermott and Shaffer found that students often reason locally and sequentially instead of holistically about a circuit.[5] This means that they only focus on where a change was made, rather than acknowledge that that change may influence the circuit at a different point. In other words, they fail to *combine* the different laws that govern a circuits' behavior. An example is when students are asked what happens to the brightness of a bulb in series with two others in parallel when one of the parallel bulbs is removed. They often think there will not be any change. Various other studies also observed these misconceptions with their students.[6,7]

Cohen, Eylon and Ganiel found that students tend to reason based on current instead of voltage or potential, which is the underlying reason for students' sequential reasoning.[5,8] A consequence is that some students have problems with an open switch: they think there cannot be a potential difference across an open switch, since "there is no current, hence no voltage."[23] This student applies Ohm's law instead of using Kirchhoff's voltage law (KVL). This type of reasoning will be referred to as 'current-based reasoning' (CBR) in the rest of the paper.

Although most of the research mentioned concerns resistive circuits, there have also been some studies on student understanding of capacitive circuits, mainly on charging and discharging capacitors in a DC situation.[3,7,9,10]

When switching from DC to AC, many of the issues found for DC circuits remain, but also new mathematical and conceptual difficulties arise. Students sometimes ignore or misinterpret KVL.[11] Others think the voltage varies spatially along a wire instead of in time.[3] A final, but important, problem students have is that they do not appreciate the frequency-dependency of the circuit and have difficulties with phase behavior.[7] A problem students have with phase shifts is that because of them, one cannot simply add amplitudes (or RMS values) algebraically.[12–14] Another issue is that they sometimes do not realize there is a phase shift between current through and voltage across the same component, but think there is one between "either quantity relative to some other current or voltage in the circuit."[11]

The research on student understanding of RC circuits with AC input signals is so far limited and usually focused on the phase shift between current and voltage.[11] One question of the AC/DC Concept Inventory asks about a high-pass RC filter (HPF), where students often mistake it for a low-pass filter (LPF) or do not appreciate the frequency dependent behavior of the circuit at all.[3]

When studying circuits in the frequency domain, Bode plots are an important tool. These plots show the gain of the circuit (output voltage over input voltage) in decibel (dB) as well as the phase shift of the output signal with respect to the input signal as a function of frequency. The frequency axis is shown on a logarithmic scale to allow for a wider frequency range. An example of the gain portion of the Bode plot of a high-pass filter is in Fig. 4. Bode plots are widely used in electronics and system theory to visualize the behavior of a filter or amplifier. In this paper, only the gain plot is discussed and any mention of 'Bode plot' in the remainder refers to the gain plot unless indicated otherwise. There has been some research in this field, mostly from a system-theory point of view.[15–17] The focus of this earlier research is on the mathematical aspects of transfer functions (e.g. its poles and zeros) in relation to the shape of the Bode plot itself (e.g., a 20dB decrease and increase in the slope, respectively), regardless of a specific context.

III. CONTEXT AND METHODS

With the exception of one question in the AC/DC Concept Inventory, none of the studies mentioned in section II look at circuits in terms of filters. Most focus on DC voltages and currents, usually in steady state or in the time domain, and describe student problems with basic circuit laws (Ohm's and Kirchhoff's laws for example). In electronics courses however, circuits are discussed from a 'filter' point of view, namely as systems having an input and output voltage. The analysis (and construction) of circuits focuses on the relationship between input and output and is almost always done in the frequency domain, using tools such as Bode plots. In Belgium, a typical engineering curriculum has a physics course in the first year, in which RC circuits are introduced via charging and discharging a capacitor with a DC voltage. In the second year, these circuits are the topic of the first lectures (and labs) of an introductory electronics course, now from a 'filter' point of view: the inputs are now AC signals and the frequency-dependent behavior of the circuit is studied.

In this paper, we aim to verify whether or not engineering students studying electronics recognize an RC circuit as a filter. In addition, we want to study how well they understand or apply basic circuit laws and to what extent problems with DC circuits discussed in literature carry over to AC circuits. Therefore, the focus of this paper is on two aspects of student understanding of passive first order RC filters that have not been studied before and we will answer the following research questions:

- How do students understand the influence of an AC versus DC input signal on the magnitude of the output signal for a given passive first order low-pass RC filter (LPF)?
- How do students understand the influence of the components (resistor and capacitor) of a passive first order high-pass RC filter (HPF) on the magnitude of the output signal for a given input signal?

A. Questionnaire

To answer both research questions, an open-ended questionnaire was developed, in which two questions related to first order RC filters were included. Three more questions were related to signals and are discussed in a separate paper.[18] The formulation of these questions was inspired by findings from 11 interviews in the spring semesters of 2011 and 2012 with second year engineering students who followed an introductory electronics course and who completed a laboratory on RC-filters a month prior to the interview.[19] These interviews aimed to detect students' conceptual problems with first order RC filters, if any. Various ideas and misconceptions surfaced, including several encountered in literature. The main problems encountered were the following:

- Problems with potential difference, e.g. measuring input and output voltage across the same capacitor as the interviewed student in Fig. 1 did;
- Current-based reasoning [when discussing an LPF with a DC input] *"That's an open switch, so there is no current flowing through the circuit and then there is no voltage across [the output] anymore."* ;
- Failure to sketch the circuit of a filter and/or uncertainty about the type of filter sketched;
- Problems using a Bode plot: students could only sketch the Bode plot corresponding to circuits they had seen during the lecture but failed to draw a qualitative sketch of e.g. a 'filter that attenuates low frequency signals instead of high frequency ones';
- Problems relating the input signal to the output signal, e.g. omitting the phase shift between both or changing the shape of a simple sine wave;
- Failure to assess the effect of changing a component in the circuit.

To verify to what extent the problems found during the interviews were also prevalent more generally, a survey was developed and administered to a group of second year engineering technology students. The questions are in Sections IV A and V A. To force the students to think conceptually rather than fill in a formula, the questions contrasted several qualitative situations and asked students to compare the output voltage of the different situations. Since the main interest was in the approach students used and the extent to which each approach was successful, the questions explicitly asked for an explanation of the given answer. Both questions can be answered by either using circuit laws or by recognizing the circuit as a filter and building a reasoning from there. This approach made it possible to verify which approach is more popular as well as which is more successful. Additionally, the specific nature of the mistakes can be documented.

The first question showed three identical low-pass filters (LPF) with different input signals: 1V DC, 10V DC, and 1V AC. The students were asked to rank the circuits according to the output voltage. The full question is in section IV A, with a discussion of the answer in IV B. As opposed to keeping the circuit constant and changing the input signal, the second question showed four circuits configured as high-pass filters (HPF), each with the same (AC) input signal. The first filter had component 'values' R and C, where the next ones had one or both components doubled or halved, depending on the questionnaire. The exact question is in section V A, while the answer is discussed in section V B. Although all questions are printed in English, they were originally asked in Dutch and the students also answered them in Dutch.

B. Participants and educational context

All participants in both the interviews and the questionnaire were second year engineering technology students, spread across three campuses in Belgium. In the first year, they attended an introductory physics course that included an introduction to electricity and circuit laws. This paper is about their introductory course in electronics, consisting of traditional lectures and lab sessions, both taught in Dutch. At one campus, this course was taught as part of the general engineering curriculum in the first semester of the second (bachelor) year, while at both other campuses, it was taught in the second semester of the second year to the students who chose to major in electronics engineering. Typically, the labs focused on a specific topic covered during one or more earlier lectures. One of those lab sessions was about first order passive RC filters. The (Dutch) questionnaire was administered in the 2012-2013 academic year, once just before entering the lab and a second time approximately 1 month after the lab. Since there was no significant change of the answers between the pre- and post-lab results (not in the number of correct answers, verified using a binomial test, nor in the distribution of explanations or answers, verified using a χ^2 test, neither even at the $p = 0.05$ level), the remainder of the article is about the post-lab results only. Similarly, there were no significant differences between the different campuses, so the results presented here are valid for all three campuses. 156 students filled in the questionnaire after the laboratory. Neither the pre-test nor the post-test was mandatory or counted for any credit, but there were no students who chose not to participate at any of the campuses.

C. Analysis

Student answers were analyzed in two ways: the first focused on the *ranking* the students provided so as to determine whether or not there was any pattern there. Then, the *explanations* of the students were categorized, in order to gain a nuanced understanding of what aspects students understand reasonably well and what aspects are problematic to them. Our interest is in reasoning patterns the students use, not only in the classification of answers in terms of right and wrong. The exact categories and their origin are described in sections IV C and V C for the low-pass and high-pass filter question respectively. To establish the validity of the classification of the answers, the third author analyzed a random subset of the data ($N = 37$, 23%) and Cohen's κ was used to determine the inter-rater reliability. The result is reported for both questions.

IV. ROLE OF INPUT SIGNAL FOR A LOW-PASS FILTER

A. Question

Below [see Fig. 2] are **3 identical circuits**. A **different input signal** is applied to each one. After some time, the output signal is measured. Sort the circuits according to the **maximum of the output voltage** from largest to smallest. Indicate explicitly if the output voltage is zero or if two output voltages are equal. **Explain your answer!** [Emphasis in the original]

B. Correct answer

The correct answer is that B>A>C. This can be found in several ways. The first is to recognize the circuit as a low-pass filter, which will allow DC signals to pass undisturbed, but which will attenuate AC signals. A second approach is to use a voltage divider (or circuit laws in general) by replacing the capacitor with its impedance ($Z_C = \frac{1}{j\omega C}$). Using a voltage divider results in $v_{out} = v_{in}\frac{Z_C}{R+Z_C}$. Replacing the (angular) frequency in the formula for Z_C by zero for the DC signal will lead to a correct answer where the DC output signal is equal to the input signal. For AC, the output will be lower, regardless of the specific values of all the parameters. A last, related approach is to replace the capacitor by a short for AC signals (equivalent to an infinitely high frequency) and by an open circuit for DC signals. Although this is not an exact solution (the frequency will be finite in any practical application), this approach is a useful way to quickly gain a qualitative picture of the situation. If Kirchhoff's voltage law (KVL) is applied correctly after replacing the capacitor with a short in AC and an open switch for DC, one finds that the output voltage will be equal to the input voltage for DC signals while for AC it will be zero.

C. Analysis

As mentioned before, the student answers were categorized in two different ways: the first focused on the *ranking* of the circuits and the other on the *explanation* given. The former resulted in 5 specific rankings that accounted for over 80% of all student answers, with the others either giving a different answer or leaving the question blank. These 5 rankings as well as the explanations used by the students are shown in Table I.

The categorization of the explanations the students gave was not done based on a pre-defined set of categories. Instead, the categories were built bottom-up from the students' answers. The first author went through a first set of answers, marking potential categories using paper and pencil. After going through this first 'training' set using this approach, the most common categories were written down. Then, the answers of a second 'validation' set were assigned to these categories. There were not many answers that did not fit in any of the original categories, therefore all data were analyzed using these categories. A final validation of the categories was done by another author categorizing a random subset of answers independently using the same set of categories. The inter-rater reliability was verified using Cohen's κ, which was 0.88, indicating near perfect agreement[20]. The categories were the following:

- *Filter*: The student recognizes the circuit as a filter and builds his/her reasoning from there.
  One student gave the following (correct) answer: "*Is an LPF (low-pass) and so DC will pass easier and 10V DC > 1V DC and so as last C, 1V.*"
  However, some students do recognize the circuit as a filter, but think it is an HPF: "*A and B are high-pass so do not allow direct current to pass. C is also high-pass so will allow alternating current to pass.*"

- *Open circuit/short*: The student replaces the capacitor by a short for input C (AC) and by an open circuit for inputs A and B (DC). By (implicitly) applying KVL, the student arrives at a correct answer. While it is possible to arrive at an incorrect answer by replacing the capacitor by a short for DC signals and an open circuit for AC signals, none of the students in our study made this mistake.
  An example of a typical answer is the following: "*B,A: capacitor on DC = open circuit ⇒ source voltage across the resistor is across V_{out}. C: $V_{out,V} = 0$ because on AC a capacitor is a short, across which there is no voltage.*"

- *Current-based reasoning (CBR)*: Here the student uses the same approach as before, replacing the capacitor by an open circuit in DC. He correctly states there is no current, from which, however, he concludes there is no voltage either. There is often no mentioning of the AC situation. This results in the incorrect answer $C > A = B(= 0)$. A typical answer was: "*In DC* $\rightarrow$ *no current through capacitor; 1V and 10V DC* $\rightarrow$ $V_{out} = 0V$*;* $C > A = B$*.*"

- *Voltage divider*: Some students (implicitly) use the formula for a voltage divider ($V_o = \frac{Z_c}{Z_r + Z_c} V_i$) and replace the capacitor by the formula for its impedance ($Z_c = \frac{1}{j\omega C}$).
  An example of the (implicit) use of a voltage divider was the following: "*[A and B] Low f so C has voltage of the source. [C] large f so voltage divides* $\rightarrow$ *so less than 1V!*"

- *AC=DC*: The student does not distinguish between AC and DC input signals and ignores the circuit, resulting in an incorrect answer.
  One typical answer was: "$B > A = C$ *input voltage in B is bigger, so output voltage also bigger.*"
  Another student wrote the following explanation: "$B > A = C \rightarrow$ *DC output voltage of A and C are equal, they are only influenced by the resistor and not by C.*"

- *RMS*: In this answer, the student ignores the circuit but does make a distinction between the AC and DC input signals by looking at the RMS-value of the signals. This most often results in a correct answer. This category also includes students who say that "*average of the AC signal is zero*," that "*the DC signal does not have an amplitude*" or similar incorrect answers that ignore the circuit but focus on the difference between an AC and DC signal.
  One example is the student who wrote: "*Source of 1V AC can be replaced by a DC source of* $\frac{1}{\sqrt{2}} V$"

- *Other*: Any other explanations included by the students.

- *No explanation*: This category contains students who did give a ranking of the different circuits, but did not include an explanation with it.

- *Blank*: The students who did not provide any answer and left the question blank.

D. Results

The results in Table I show the number of students for each combination of ranking and explanation, as well as the totals for each categorization (by ranking and explanation).

There is a big group of students who give a ranking but do not provide any explanation (over 50%). However, there is no significant difference in the distribution of rankings given by students who include an explanation and those who do not according to a χ^2-test (even at the $\alpha = 0.05$ level). Therefore, it is reasonable to suppose that the explanation underlying the answer of a student who did not provide an explanation is similar to the reasoning of those who do.

The results in Table I indicate that around 50% of the students gave a correct answer to this question. However, of the 39 students who also explained their (correct) answer, 12 students (over 30%) gave an *incorrect* explanation, such as the RMS-based reasoning and various explanations categorized as 'Other'. If we assume that also 30% of the 39 students who gave a correct answer without writing down an explanation arrived at it using a wrong approach, only one third of all students (not counting blanks) actually arrived at a correct answer by using a correct approach. These students who do give a correct explanation, use different methods, including recognizing the LPF, replacing the capacitor by a short or open circuit in AC and DC situations respectively and using a voltage divider. It is not clear what kind of reasoning the students who gave no explanation used, although the distribution is most likely similar to that of those who did include one.

When studying the distribution of the other, incorrect, rankings, two have a clear correlation with a specific explanation. The first is the ranking $C > A = B$: all but one student who provided an explanation with this answer used current-based reasoning. It is therefore fair to say that the students who used this ranking but did not provide an explanation probably made the same mistake. The second category are the students who do not distinguish between AC and DC input voltages, arriving at the conclusion that $B > A = C$. All students who arrived at this ranking and provided an explanation, fit in this category. Again, the students who arrived at the same conclusion without explaining it, probably made the same mistake.

A final conclusion has to do with both types of (correct) answer categories: using a filter-based approach or using an approach based on circuit theory (e.g. open circuit/short or voltage divider, but also current-based reasoning). The former approach is less popular: 13

students (21% of those who provide an explanation) use it as opposed to 37 (61%). However, the former approach is more successful with a success rate of nearly 80% (10 students) compared to one of barely 50% for the latter.

V. COMPONENT VARIATION FOR A HIGH-PASS FILTER

A. Question itself

The circuits below [see Fig. 3] all have the **same AC voltage** (finite amplitude and finite frequency) as input signal. However, the **values of the resistor and capacitor are different** in every circuit. Sort the circuits according to decreasing **amplitude of the output voltage** (highest to lowest). Explicitly indicate if the output voltage is zero or if the output voltage in two situations is equal. **Explain your answer!**
[Emphasis in the original]

B. Correct answer

The correct answer is that $B > C = D > A$. This answer can be obtained in various ways, the first of which uses the fact that this circuit is a high-pass filter (HPF). As shown in Fig. 4, this can be most easily seen by using the Bode plot of a first order high-pass RC filter. When looking at circuits A and D for example, one can deduce that the cut-off frequency of circuit D will be half that of circuit A ($f_c = \frac{1}{2\pi RC}$). Consequently, the Bode plot of circuit D will 'shift to the left' with respect to that of circuit A, which in turn leads to a higher gain at a certain frequency f for circuit D. This higher gain means that for a constant input amplitude, the output amplitude of circuit D will also be higher than that of circuit A. Using the same approach for all 4 circuits, the Bode plots for circuits C and D coincide, while B will be shifted even more to the left.

A second approach is that there is no current in the DC circuits, so there is no voltage drop across the resistor, resulting in the entire input voltage being across the output terminals (across the capacitor). However, there is a current in the AC circuit, so there is a voltage drop across the resistor, resulting in a lower voltage across the capacitor. Not only that, but doubling the resistor has the same effect as doubling the capacitor. This again leads to the conclusion that circuit B will have the highest output voltage (doubling both the resistor and the capacitor results in an even higher output voltage), followed by circuit C and D and finally circuit A with the lowest output voltage.

It is also possible to explicitly use classical circuit laws (Kirchhoff's laws and Ohm's law) in order to arrive at the same conclusion. The voltage divider approach discussed earlier is essentially a shortcut of this approach.

C. Analysis

As with the low-pass filter question, the answers to this question were also categorized based on both the ranking the students gave and their explanation for that ranking. However, since there are too many possible rankings of 4 circuits (75 when assuming they are ranked from highest to lowest and equality is possible), the answers themselves were analyzed by looking at the relationship between specific *pairs* of circuits shown in Fig. 3. The reason for this is that there were too many different rankings used by the students that were all relatively rare. For example, there were only 8 students (5%) who gave the correct answer. Therefore, it is interesting and very useful to have a look at what aspect the other students *did* understand, even though they did not manage to give a fully correct answer. In other words: to find out what the wide variety of other answers have in common. The first aspect is the effect of changing the resistor on the output voltage, done by comparing the 'standard' circuit A to circuit D, in which the resistor is changed. The second is the effect of changing the capacitor, similarly done by comparing circuits A and C. The third comparison was the effect of changing both the capacitor and resistor (circuit B compared to A), while the fourth and last one compares the effect of only changing the resistor (circuit D) to that of only changing the capacitor (C). For each of those pairs, there are 4 possible answers: is greater than, is smaller than, is equal to and no information. The latter usually indicates a blank answer, but can also mean that there was an answer given, but without information about a certain pair, for example answering "B>A," which does not contain any information about circuits C or D.

To explain their answers, students used various strategies. The different explanations are listed below. Again, these categories were found bottom-up from the data, using the same approach used for the low-pass filter question. The Cohen's κ for the eventual categories used was 0.75, still indicating a substantial agreement between both raters[20].

- *Filter*: By recognizing the circuits as high-pass filters and using their cut-off frequency

($f_c = \frac{1}{2\pi RC}$), one can deduce that the higher R and/or C, the higher the output voltage will be for a given frequency of the input signal. An example of a correct answer using this approach is the student who wrote that "*HPF:* $\omega_c = \frac{1}{RC}$*, R >> [increases, so]* ω_c *small; C >> [increases, so]* ω_c *small*" while adding a sketch of the Bode plot of a high-pass filter.

However, this approach also went wrong when students think that, for example, the output is proportional to the cut-off frequency or make a mistake in the formula for the cut-off frequency. An example of the former is this answer: "*high-pass filter,* $A = \frac{1}{\sqrt{\omega_c}}$ $\rightarrow$ *if* C *is smaller* $\rightarrow \omega$ *bigger* $\rightarrow f$ *is bigger* $\rightarrow$ *more passes and A bigger.*"

- *Voltage divider*: Using the formula for a voltage divider ($v_{out} = \frac{Z_R}{Z_C+Z_R} v_{in}$) the correct answer follows readily by using the impedance of the capacitor, which is inversely proportional to its capacitance ($Z_c = \frac{1}{j\omega C}$). Some students however, made a mistake using this approach by thinking that the capacitor impedance is proportional to its capacitance.

  One student wrote down the exact formula and arrived at the correct conclusion: "$V_o = V_i \left(\frac{R}{R+\frac{1}{j\omega C}} \right)$ *so if RC small,* V_o *small. C and D are equal.* $B > C = D > A$*.*" Another one made a mistake by using the capacitance instead of impedance: he wrote $V_o = V_i \left(\frac{R}{R+C} \right)$ for every circuit and (consistent with the formula) arrived at the conclusion that $D > A = B > C$.

- *Circuit laws*: It is possible to use classic circuit laws such as Ohm's law and Kirchhoff's laws to arrive at a correct answer without using the 'shortcut' of a voltage divider. Some students attempted this approach, but none were successful.

  An example of such an attempt is by a student who used (only) Ohm's law: "$X_C = \frac{1}{2\pi C}$; $C > A = B > D$. *If you double C, the impedance changes, it becomes twice as small. If you double R, the impedance becomes twice as big.* $R = \frac{U}{I} \rightarrow U = R \cdot I$*.*"

- *R matters more*: Some students think that only the resistance of the resistor matters, or that it matters more than that of the capacitor. The reasoning for this varies, but the following three explanations are typical:

  - The first is related to so-called local reasoning, saying that the output voltage only depends on the resistor simply because it is closer to where the output voltage is measured. What happens 'far away' from there is ignored.

An example is the following answer: "$B = D, C = A$ *because of Ohm's law. More voltage across resistor and capacitor in AC is short.*" Note that this student also did not realize that replacing the capacitor by a short would lead to an equal input and output.

  - A second type were those who ignored the capacitor because it would not influence the amplitude: "$A = C > D = B$ *capacitor does not influence the amplitude of the output voltage.*"

    One student further explained that "*The capacitor causes a phase shift, resistor regulates* V_{out}*. So R big* $\rightarrow$ *V big.*"

  - A last explanation was that the resistor and capacitor both have an influence, but that the capacitance of the capacitor mattered less:

    One student called circuits with a higher capacitor 'better filters': "*Bigger output impedance for [circuits] B and D for AC* $\rightarrow$ *bigger* V_{out}*. Better filter* $\rightarrow$ *bigger* V_{out}*. A: HPF, B: double HPF, C: double HPF, D: HPF* $\rightarrow B > D > C > A$."

    Another one had a different reason: "*Circuit B has the biggest amplitude. This is because both C and R are doubled. Then comes circuit D because R has more influence than C (is a very small number). Then circuit C and then A.*" This student argued that the total impedance of the circuit is what matters and also made the mistake of thinking that the capacitor impedance is proportional to its capacitance.

- *C matters more*: As with the previous explanation, this means that a student thinks only the capacitor matters or that it matters more than the resistor. There was no clear reason why students did this as their explanations varied greatly.

  Some just wrote that only the capacitor mattered: "$B = D, A = C$ *resistor has no influence on the amplitude, capacitor does.*"

  Others seemed to think via the current, but ignored that the resistor would also have an influence on the current. One wrote: "*the bigger C, the bigger i through C, the less current through R and the smaller* v_{out}*.* $D > A > B > C$." This is an example of a misconception about current being 'used up' in 'earlier' circuit elements, leaving less current available for the one where measurements take place, combined with local reasoning.

One student replaced the capacitor by a short, but also considered the capacitor charging and discharging: "$B = C - D = A$ *Because first the capacitor is a short and all of the voltage will be across the resistor. Then when it discharges, there is again the full voltage across the resistor. Because of this, the resistor does not matter, only the capacitor.*"

- *Other*: Any other explanations included by the students.
- *No explanation*: This category contains students who did give a ranking of the different circuits, but did not include an explanation with it.
- *Blank*: The students who did not provide any answer and left the question blank.

D. Results

The results are in Tables II and III. The first shows the number of students for each combination of ranking and explanation, as well as the totals for each categorization (by ranking and explanation). The second shows a cross table of all possible combinations of the different rankings of the students who gave a full answer (132 or 85%). There are some interesting conclusions that can be drawn from these tables, which are explained in more detail below:

- Most students know that changing the capacitor and/or the resistor will *influence* the output voltage;
- Many students do not know the *direction* of this influence;
- Many students do not know that the effect of doubling the resistor and capacitor is equal in *magnitude*.

Before delving deeper into the reasons behind the conclusions stated above, it may be interesting to clarify some of the more obvious results from Table II. The first is the low number of students who rank different pairs correctly, ranging from 43% when comparing circuits A and D to only 12% for the comparison of C and D. A second is the low number of students who provide an explanation: 37%. A reason for this could be that it is cognitively hard to rank 4 different circuits in which two parameters are changed simultaneously (as mentioned, there are 75 possible ways to rank the circuits from highest to lowest when allowing for equality between circuits). However, as with the LPF-question, there is no significant difference in the distribution of the rankings given by students who give an explanation and those who do not according to a χ^2-test (even at the $\alpha = 0.05$ level). Given this lack of difference between students who explained their answers and those who did not, it is reasonable to suppose that the reasons underlying the answers of the students who did not provide an explanation are similar to the reasons of those who did.

When looking at the incorrect answers, it is clear that students know very well that changing either the capacitor or the resistor *will have an influence* on the output voltage. There are hardly any students who indicate that changing the resistor or capacitor from the 'standard' circuit A will not have any influence: 1 (<1%) saying that C=A and 6 (<5%) that D=A, respectively. However, most students do not know in what *direction* this influence will be: there are nearly as many students who think the output will decrease (64, 41%) as there are who (correctly) think it will increase (67, 43%) when comparing a circuit with a doubled resistor (D) to the 'standard' one (A). The same is true when doubling the capacitor, although here more students incorrectly think a higher capacitance will lead to a lower output voltage ($C < A$) than there are who correctly think it will lead to a higher one ($C > A$) (73 (47%) and 55 (35%) respectively).

1. Logical consistency among rankings

Although the results appear to be relatively random, Table III makes clear that the students' answers are very consistent and make logical sense. What we mean by 'logical sense' here is more than the order being possible (i.e. there are no students who say that $B > A$ and $C < A$, but still conclude that $C > B$). It refers to consistency in a students' *reasoning*. For example, there are 34 students (22%) who correctly think that doubling the resistor will increase the output voltage ($C > A$) and that doubling the capacitor will also increase the output voltage ($D > A$). Of those 34, 33 logically concluded that the circuit that has both the resistor and the capacitor doubled will have a higher output voltage than the original circuit ($B > A$). Only 1 of the 34 stated the illogical $A > B$. The latter is not contradictory in the same sense as the previous example, but it contains a different type of logical error: if doubling the resistor will increase the output voltage and doubling the capacitor will as well, then it makes no (logical) sense that doubling both will decrease the output voltage.

There is however one specific group of students who do give illogical answers in the sense described above: of the 42 students (27%) who (wrongly) think that doubling either the resistor or the capacitor will decrease the output voltage, 13 (31%) either think that doubling both will lead to an increase of the output voltage or to no change at all in the output voltage. So although over 90% of the students give a consistent (albeit most often incorrect) answer, it is unlikely that all 16 students who give an illogical answer simply made a guess: 13 (80% of this group) think that doubling the resistor or capacitor will decrease the output voltage, but still think that doubling both will either not affect the output or will result in an increase of the output voltage. Most likely, these students do not have a clear misconception, but instead use a different type of reasoning when comparing circuits C and D to circuit A than when comparing B to A.

Looking at the cross-table also shows that students can be logical when comparing circuit A to the other three, but still make mistakes when comparing circuits C (with a doubled capacitor) and D (doubled resistor) while still making logical sense. For example, of the 33 students mentioned earlier who (correctly) think that doubling the resistor and/or the capacitor will increase the output voltage, only 8 (25%) also say that doubling the resistor will have the same effect on the output voltage as doubling the capacitor (C=D). 14 of the remaining 25 think that $C > D$, which is a logical answer if one (wrongly) assumes that doubling the resistor will have a bigger effect than doubling the capacitor. Similarly, the remaining 11 think that $D < C$, which is consistent with the (again wrong) assumption that doubling the capacitor will have a bigger effect than doubling the resistor.

2. Correlating rankings with explanations

In terms of explanations the students gave, there are several conclusions that can be made. First of all, the students who thought that the resistor change and capacitor change would cancel each other out and explained their answer, all used classical circuit laws or a voltage divider. They usually made the mistake of thinking that the capacitor's impedance was proportional to its capacitance.

Secondly, 14 students (34% of those who provided an explanation) explicitly stated that the change in capacitance or resistance mattered more than the other or vice versa. However, there is no clear reason why students do this: some think doubling a component will cause an increase in output voltage while others think it will cause a decrease; some think the other component still matters but is less important while others think it has no influence at all; some cite signal properties (phase versus amplitude) as a reason while others refer to circuit laws to justify their answer. There is no clear pattern in the answers the students give using this approach, but it is a very interesting observation that deserves further investigation.

Next, it is remarkable that none of the students who used classical circuit laws without using a voltage divider managed to arrive at a correct answer. The students' explanations revealed several problems with the use of these circuit laws, including local reasoning, not knowing the impedance of a capacitor and the attribution of a higher importance to one of both components mentioned earlier. This was also observed during the interviews, with many of those students displaying similar problems.

A final observation has to do with the two types of explanation: based on recognizing the circuit as a filter and (implicitly) using classical circuit theory. In total, 9 students (22% of those who provided an explanation) explained their answer from a filter point of view. 2 of them managed to arrive at a fully correct answer. The remaining 32 students (78%) used an explanation based on circuit theory, of which only 3 managed to arrive at a correct answer (using a voltage divider approach). This indicates that the students who are using a filter approach are more successful than those using classical circuit theory (a success rate of over 20% as opposed to one of less than 10%). As the number of students using the filter-based approach is small, it is not possible to make any statements to generalize these results. Nevertheless, it is very interesting that there are so few students who managed to apply classical circuit laws correctly.

VI. DISCUSSION

In this study, we investigated students' conceptual understanding of first order RC filters in the context of an introductory electronics course. We found that students struggle to analyze basic RC filters after relevant instruction. In particular, tasks about the influence of the input signal and of different circuit components showed to be problematic. Considering that these students have already passed an earlier university level course of physics and are currently attending an electronics course, the number of students making these errors is high. In addition, relatively few students provided an explanation with their answer. The reason for this is unclear, although it could be because the questions were at the end of the questionnaire, which was limited in time (10-15 minutes) and did not count for any credit. It is therefore possible that students were less thorough or attentive answering these questions.[21,22] That being said, the consistency and patterns observed in both questions indicate that the students did answer these questions seriously and put effort into answering them as correctly as possible.

A first important finding is that students in general think about these circuits using circuit laws rather than using a filter-based approach. While both are equally correct and useful in this case, it indicates that students are still more comfortable using the laws with which they are familiar. However, well known problems using those laws persist while students who do recognize the filter tend to perform better than their colleagues. That being said, the students who use a filter-based approach in the LPF question do not necessarily do so in the HPF question or vice versa.

Second, many students made mistakes known in literature. A very clear example is the current-based reasoning, which was also observed in several other studies.[5,8] Another problem observed in earlier studies are the students who do not appreciate the frequency-dependent behavior of the circuit and think there is no difference between AC and DC input signals.[3,7] However, an interesting new finding is that some students thought the question was about the RMS value, despite there not being any mention of power in the question. This did not occur during the interviews, so it is unclear why this is triggered.

When answering the high-pass filter question, it was very clear that students realized that both changing the resistor value and the capacitor value would have an influence on the output voltage. However, most did not know that doubling either component would lead to an increase in output voltage. In other words, they did not know the *direction* of the influence. Similarly, many did not know that both effects would be the same: doubling the resistor would result in the same output voltage as doubling the capacitor. The reasons for this are made more clear by the explanations provided with the student answers. Most often, students exhibited problems with classical circuit laws. One of the most important ones was that they had problems when analyzing circuits as voltage dividers, making three kinds of mistakes when doing so. The first was to think (implicitly) that the impedance of the capacitor is proportional to its value. The second is probably related to the local reasoning problem described in earlier studies: because the output voltage is measured across the resistor, students think that the resistor value has more influence than the capacitor value on the output voltage.[5–7] A final problem they have is that some students think the *capacitor* has a bigger influence than the resistor.

In general, several observations have been made that deserve further investigation:

- Even after lecture and lab instruction about filters, students tend to think about circuits using classical circuit laws rather than using a filter-based approach. Does this preference decrease over time? Does it depend on major (e.g. physics students using a different approach than engineering students)?;

- Despite passing an introductory physics course and studying electronics, students have problems applying those circuit laws when AC signals and frequency-dependent components are involved. Are the problems students have in AC caused by underlying misconceptions about basic laws in DC or do they see AC circuits as a completely different realm in which classical circuit laws are no longer valid?

- Some students think resistors have more influence than capacitors or vice versa. What is the cause of this?

- Some students completely ignore the frequency-dependent behavior of a circuit. Why do they ignore it?

- Most students know that both changing the resistor and capacitor will have an influence on the output of a first order RC high-pass filter, yet do not know in what direction this influence will be, nor that the influence is the same for the resistor and the capacitor. Why do they think one is more important than the other? And why do they do know that both will have an influence?

- Many students did not manage to correctly assess the difference between AC and DC input signals for a low-pass filter. Are they capable of doing so for high-pass filters?

- Similarly, most students did not manage to correctly assess the influence of a changed component on the behavior of a high-pass filter. Are they able to do so for a low-pass filter?

Although many misconceptions about circuit laws are discussed in literature, current instruction does not seem to address those misconceptions: even after passing an introductory physics course and attending a lecture and laboratory session on RC filters, students still make many mistakes even when, for example, using a voltage divider. More effort should be put into a correct conceptual understanding of basic circuit laws in introductory courses because a lack thereof hampers proper understanding of subsequent concepts. This is very clear from our research, which indicates that known problems with circuit laws directly translate to problems related to first order passive RC filters even after relevant instruction. In future research, we will try to develop a laboratory to increase students' understanding of first order RC filters, taking the findings discussed above into consideration.

When summarizing the general findings of this study, it is important to be aware of its limitations. A first observation is that rather few students included an explanation with their answer. So although there is good reason to assume that the students who did not provide an explanation used a similar reasoning to those who did, this is not certain. Therefore, conclusions related to the explanations given for the students' rankings have to be treated with caution. It is, for example, possible the students who did not include an explanation did all their work mentally, but used a different approach than those who did provide an explanation. Or maybe those who did not give an explanation simply guessed, although that seems very unlikely given the data. Second, we also decided to incorporate results that only provided an incomplete ranking (e.g. only stating that 'A=B' without mentioning the other circuit(s)), especially when analyzing the high-pass filter question. While these results are certainly valuable and have to be taken into account, it is also possible to treat them separately. A final aspect of filters not covered in this study is the *phase shift* they cause between input and output signals. Although we investigated whether or not students understood the influence of components (for an HPF) or AC opposed to DC input signals (for an LPF), more research is needed to understand whether or not students really understand all aspects of filters, including the phase shift.

ACKNOWLEDGEMENTS

The authors would like to thank all students who volunteered for the interviews, as well as those answering the survey(s). Our thanks also goes to the teachers who gave us some of their course-time to conduct the surveys. Last but not least, we are grateful to Christian Kautz, MacKenzie Stetzer and Wim Van Dooren for valuable discussions about both the interviews and the surveys.

---

[*] pieter.coppens@kuleuven.be

[†] permanent address: Celestijnenlaan 200C - bus 2406, 3000 Leuven, Belgium

[1] B. M. Zavala, "Teaching electricity in elementary, middle and high school: Some real life examples," Master, Iowa State University, 2008. [Online]. Available: https://books.google.be/books?id=kI{_}oaixXVxkC{&}printsec=frontcover{#}v=onepage{&}q{&}f=false

[2] V. Picciarelli, M. Di Gennaro, R. Stella, and E. Conte, "A Study of University Students' Understanding of Simple Electric Circuits Part 2: Batteries, Ohm's Law, Power Dissipated, Resistors in Parallel," *European Journal of Engineering Education*, vol. 16, no. 1, pp. 57–71, 1991. [Online]. Available: http://www.tandfonline.com/doi/abs/10.1080/03043799108939504

[3] D. Holton, A. Verma, and G. Biswas, "Assessing student difficulties in understanding the behavior of AC and DC circuits," in *ASEE*, 2008.

[4] P. V. Engelhardt and R. J. Beichner, "Students' understanding of direct current resistive electrical circuits," *American Journal of Physics*, vol. 72, no. 1, pp. 98–115, 2004. [Online]. Available: http://link.aip.org/link/AJPIAS/v72/i1/p98/s1{&}Agg=doi

[5] L. C. McDermott and P. S. Shaffer, "Research as a guide for curriculum development: An example from introductory electricity. Part I: Investigation of student understanding," *American Journal of Physics*, vol. 60, no. 11, pp. 994–1003, 1992.

[6] A.-K. Carstensen and J. Bernhard, "Threshold concepts and keys to the portal of understanding," in *Threshold concepts within the Disciplines*, 2007.

[7] D. L. Holton and A. Verma, "Designing Animated Simulations and Web-based Assessments to Improve Electrical Engineering Education," in *Web-Based Engineering Education: Critical Design and Effective Tools*, D. Russell and A. Haghi, Eds. IGI Global, 2010, pp. 77–95. [Online]. Available: http://works.bepress.com/douglas{_}holton/5/http://services.igi-global.com/resolvedoi/resolve.aspx?doi=10.4018/978-1-61520-659-9

[8] R. Cohen, B.-S. Eylon, and U. Ganiel, "Potential difference and current in simple electric circuits: A study of students' concepts," *American Journal of Physics*, vol. 51, no. 5, p. 407, 1983. [Online]. Available: http://link.aip.org/link/?AJP/51/407/1{&}Agg=doi

[9] D. P. Smith and P. V. Kampen, "A qualitative approach to teaching capacitive circuits," *American Journal of Physics*, vol. 81, no. 5, p. 389, 2013. [Online]. Available: http://link.aip.org/link/AJPIAS/v81/i5/p389/s1{&}Agg=doi

[10] S. Pulé, "Students' versatility with resistor-capacitor circuits," *International Journal of Electrical Engineering Education*, vol. 49, no. 4, pp. 419–443, oct 2012. [Online]. Available: http://manchester.metapress.com/openurl.asp?genre=article{&}id=doi:10.7227/IJEEE.49.4.5

[11] C. H. Kautz, "Development of Instructional Materials to Address Student Difficulties in Introductory Electrical Engineering," in *SEFI Annual conference Lisbon (World Engineering Education Flash Week)*, J. Bernardino and J. C. Quadrado, Eds., 2011, pp. 228–235.

[12] J. Bernhard and A.-K. Carstensen, "Learning and teaching electrical circuit theory," in *Physics Teaching in Engineering Education*, Leuven, 2002.

[13] ——, "Understanding phase as a key concept in physics and electrical engineering," in *SEFI Annual conference*, no. September, Leuven, 2013, pp. 16–20.

[14] A. P. Mazzolini, S. Daniel, and T. Edwards, "Using interactive lecture demonstrations to improve conceptual understanding of resonance in an electronics course," *Australasian Journal of Engineering Education*, vol. 18, no. 1, pp. 69–88, 2012. [Online]. Available: http://www.tandfonline.com/doi/abs/10.7158/D12-004.2012.18.1

[15] A.-K. Carstensen and J. Bernhard, "Bode plots not only a tool of engineers, but also a key to facilitate students learning in electrical and control engineering." in *Physics Teaching in Engineering Education*, Leuven, 2002.

[16] K. E. Wage and J. R. Buck, "The continuous-time signals and systems concept inventory," *IIEEE International Conference on Acoustics Speech and Signal Processing*, pp. IV–IV, 2002. [Online]. Available: http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=1004823

[17] K. Wage, J. Buck, and M. Hjalmarson, "Analyzing Misconceptions using the Signals and Systems Concept Inventory and Student Interviews," in *2006 IEEE 12th Digital Signal Processing Workshop & 4th IEEE Signal Processing Education Workshop*. IEEE, sep 2006, pp. 123–128. [Online]. Available: http://ieeexplore.ieee.org/xpls/abs{_}all.jsp?arnumber=4041044http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=4041044

[18] P. Coppens, J. Van den Bossche, and M. De Cock, "Student understanding of phase shifts, frequency and Bode plots," *International Journal of Electrical Engineering Education*, 2016.

[19] P. Coppens, M. De Cock, and C. H. Kautz, "Student Understanding of Filters in Analog Electronics Lab Courses," in *40th SEFI (Société Européenne pour la Formation des Ingénieurs) Annual Conference:Engineering Education 2020: Meet the Future*, A. Avdelas, Ed. Thessaloniki: SEFI-Société Européenne pour la Formation des Ingénieurs, 2012, pp. 196–197. [Online]. Available: http://www.sefi.be/conference-2012/Papers/SEFIBookcomplete.pdf

[20] J. R. Landis and G. G. Koch, "The measurement of observer agreement for categorical data." *Biometrics*, vol. 33, no. 1, pp. 159–174, 1977.

[21] M. Galesic and M. Bosnjak, "Effects of questionnaire length on participation and indicators of response quality in a web survey," *Public Opinion Quarterly*, vol. 73, no. 2, pp. 349–360, 2009.

[22] J. A. Krosnick, A. L. Holbrook, M. K. Berent, R. T. Carson, W. M. Hanemann, R. J. Kopp, R. C. Mitchell, S. Presser, P. A. Ruud, V. K. Smith, W. R. Moody, M. C. Green, and M. Conaway, "The impact of no opinion response options on data quality: Non-attitude reduction or an invitation to satisfice?" *Public Opinion Quarterly*, vol. 66, no. 3, pp. 371–403, 2012. [Online]. Available: http://poq.oxfordjournals.org/content/66/3/371.full.pdf+html

[23] Quote from a student answer in the authors' study

TABLE I. Results of low-pass filter question with varying input signal. Number of students who give a certain answer and give a certain explanation with that answer.

	B>A>C*	C>A=B	B>A=C	C>B>A	B>C>A	Other	Blank	Total	Total %
Filter†	10	1	0	1	0	1	0	**13**	**8**
Open circuit/Short†	8	0	0	0	0	0	0	**8**	**5**
Voltage divider†	6	0	0	0	0	0	0	**6**	**4**
CBR	0	4	0	0	0	1	0	**5**	**3**
AC=DC	2	0	8	0	0	1	0	**11**	**7**
RMS	5	0	0	0	2	0	0	**7**	**4**
Other	8	0	1	0	2	0	0	**11**	**7**
No explanation	39	8	6	5	13	15	0	**86**	**55**
Blank	0	0	0	0	0	0	9	**9**	**6**
Total	**78**	**13**	**15**	**6**	**17**	**18**	**9**	**156**	
Total %	**48**	**8**	**10**	**4**	**11**	**13**	**6**		

*Correct answer

†Correct explanation if applied correctly

(a) (b)

FIG. 1. An example of an interviewed student not understanding potential difference. When asked to sketch 'a filter', the student sketched the circuit of Fig. 1a, but did not realize that the input and output voltages would be equal, nor that the filter needs a resistor. When probed and encouraged by the interviewer, he added a resistor to his circuit as shown in Fig. 1b. When asked to clarify where he would measure the output voltage now, he added the two dashed lines in the same figure, clearly indicating he did not realize that the input and output voltage of his circuit would always be equal.[19]

FIG. 3. Circuits high-pass filter question

FIG. 4. Explanation of high-pass filter question. The solid line represents the gain portion of the Bode plot of a circuit with a resistor with resistance R and a capacitor with capacitance C, configured as a high-pass filter. Doubling the resistor (or capacitor) will result in a curve that is shifted to the left (dashed line), with respect to the original one. At a certain frequency f, the gain (so also the output voltage for an equal input voltage) will then be higher for the circuit with the increased resistor.

FIG. 2. Circuits low-pass filter question

TABLE II. Results of high-pass filter question with varying component values. Number of students who give a certain ranking and give a certain explanation with that answer.

	RC*-2*RC				***RC*-*R*2*C***				***RC*-2*R*2*C***				**2*RC*-*R*2*C***					
	D>A*	**D=A**	**D<A**	**No info**	**C>A***	**C=A**	**C<A**	**No info**	**B>A***	**B=A**	**B<A**	**No info**	**C=D***	**D>C**	**C>D**	**No info**	**Total**	**Total %**
Filter†	4	0	5	0	5	0	4	0	6	0	3	0	3	4	2	0	**9**	**6**
Voltage divider†	9	0	5	0	7	0	7	0	3	6	5	0	4	6	4	0	**14**	**9**
Circuit laws†	0	0	3	1	1	0	2	1	1	2	1	0	0	1	2	1	**4**	**3**
***R* matters more**	5	0	3	0	3	3	2	0	5	0	3	0	0	5	3	0	**8**	**5**
***C* matters more**	2	1	2	1	4	1	1	0	4	0	2	0	0	1	5	0	**6**	**4**
No explanation	47	0	46	5	35	2	57	4	40	15	39	4	11	55	30	2	**98**	**63**
Blank	0	0	0	17	0	0	0	17	0	0	0	17	0	0	0	17	**17**	**11**
Total	67	1	64	24	55	6	73	22	59	23	53	21	18	72	46	20	**156**	
Total %	43	1	41	15	35	4	47	14	38	15	34	13	11	46	30	13		

*Correct answer

†Correct explanation if applied correctly

TABLE III. Cross table of all possible ranking combinations for the students who gave a complete answer.

		B>A*			**B=A**			**B<A**				
		C=D*	**D>C**	**C>D**	**C=D***	**D>C**	**C>D**	**C=D***	**D>C**	**C>D**	**Total C↔A**	**Total D↔A**
D>A*	**C>A***	8^{+}	14^{R}	11^{C}	0	1	0	0	0	0	34	67
	C=A	0	3^{+}	0	0	0	0	0	0	0	3	
	C<A	0	8^{R}	0	0	10^{+}	0	0	12^{C}	0	30	
D=A	**C>A***	0	0	1^{+}	0	0	0	0	0	0	1	1
	C=A	0	0	0	0^{+}	0	0	0	0	0	0	
	C<A	0	0	0	0	0	0	0	0	0^{+}	0	
D<A	**C>A***	0	0	7^{C}	0	0	4^{+}	1	1	6^{R}	19	64
	C=A	0	0	0	0	0	0	0	0	3	3^{+}	
	C<A	3	2	0	1	4	3	4^{+}	16^{C}	9^{R}	42	
Total C↔D		11	27	19	1	15	7	5	29	18	132	
Total B↔A		57			23			52				

*Correct answer

+Consistent answer if influence of R and C are equal in magnitude

RConsistent answer if influence of R is more important than that of C

CConsistent answer if influence of C is more important than that of R

All other cells are inconsistent answers